The Kinematics of Simple Shear

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Abstract

The partition of a simple shear into a strain and a rotation is demonstrated to be physically unfounded, it is not helpful to understand flow. Ideal plastic simple shear is predicted to have two non-orthogonal eigendirections, the extending one is close to the bulk shear direction. The predicted eigendirections correlate well with observed fabric elements in rocks. The layering in mylonites is interpreted as a composite fabric element without mechanical significance, it does not mark a shear plane. Flow in mylonites and fluids appears laminar only at a scale larger than the one at which the deformation mechanism works.

Introduction

The understanding of deformation has been hampered by traditionalism. Since the late 18^{th} century the cause of deformation has been assumed to be stress, and the measure of deformation has been assumed to be strain. Both concepts are tensors, both have orthogonal principal axes, so their correlation must have been irresistible. Thus, the common way has been to deduce the properties of displacement – which is a vector field – from strain considerations. The approach has not led to a satisfactory understanding of plastic simple shear and the fabric it produces. But simple shear is enigmatic not only in the plastic field. In the elastic field there is the Poynting effect, i.e. a solid subjected to simple shear dilates elastic-reversibly; in the viscous field it is the existence of the Prandtl boundary layer and its reduced mass density, and the generation of turbulence. One may ask, though, why orthogonal concepts should be the right ones to understand a monoclinic problem.

The stress tensor, which is orthogonal by definition, was conceived in 1776 by Euler and worked out by Cauchy (1827), long before vector fields came into use in the 1860s which may be monoclinic. The understanding of the physics of mechanical loading was put upon entirely new tracks by the discovery of the First Law of thermodynamics in the mid-19^{th} century. However, the older concepts of deformation theory were not given up then, but the new law was adapted to fit on the already existing theories (Koenemann 2008G). This omission is at the root of the failure to understand simple shear today, because the theory has thus retained its pre-First-Law-mathematical and physical structure, leaving deformation profoundly misunderstood: elastic deformation as a physical process is not a part of conservative Newtonian mechanics in the sense of the conservative energy conservation law $E_{kin} + E_{pot} = const$, but it is a change of state in the sense of the First Law of thermodynamics $dU = dw + dq$, the energy conservation law for non-conservative processes. Euler's virtual work concept is obsolete since the discovery of the true nature of the work involved – it is change-of-state work, thermodynamic work, commonly called PdV-work. Here I take the opposite approach from the traditional one: a force vector field derived from potentials and subject to boundary conditions is assumed to be the cause of deformation; it leads directly to the displacement field; the strain may then be readily derived.

Significance of eigendirections

30 years ago the instantaneous and the finite strain directions were emphasized; more recently the concept of an eigendirection gained support, both in Euclidean and in Mohr space. Properties were assigned to eigendirections which they cannot have. Attempts have been made to identify eigendirections in the rock fabric. The question is then, eigendirection of what? Which physical term is referred to? A few explanations might be helpful.

The definition of a vector field is

$$\mathbf{Ax} = \mathbf{b} \qquad \text{(eqn.1)}$$

where $\mathbf{A}(Q)$ is a matrix or tensor and function of a point $Q(X_i)$ in a coordinate system X_i, and $\mathbf{x}$ is the position vector of a point P relative to Q. Commonly $P(x_i)$ is given in an internal coordinate system x_i with the origin Q in X_i. $\mathbf{x}$ is thus by definition a radial vector field. The operation assigns a vector $\mathbf{b}$ to the point P, as a function of the field property tensor $\mathbf{A}$ and the position $\mathbf{x}$.

The eigendirections of a field are those in which **x** and **b** are parallel such that only radial displacements are observed. In all other directions **b** has a lateral component. In 2D there is commonly a contracting, inward-directed and an extending, outward-directed eigendirection. In many engineering problems the condition that the eigendirections are mutually perpendicular is equivalent to an equilibrium condition. In bonded solids other constraints exist, and fields may or may not be orthogonal. The eigendirections must be real so that they are fixed in space.

Eigendirections are found from a physical approach through an eigenvalue routine from which a characteristic equation is derived. It has as many solutions as there are dimensions. If these solutions are real, i.e. if their derivation involves only the roots of positive numbers, they are stable, non-rotating directions. 'Non-rotating' in the present context also means that *eigendirections cannot shear*, they can only stretch or shorten. If the characteristic equation has only one solution it is said to be degenerate. A physical example would be a situation where an evolving stable condition reaches disequilibrium, or a pencil standing on its point. A degenerate characteristic equation is thus a mathematical warning sign. If the derivation of the eigenvalues involves the roots of negative numbers the resulting eigenvectors are said to be imaginary, implying external rotation and disequilibrium by definition.

Principal axes, e.g. the main axes of an ellipsoid, are found through an eigenvalue routine from a given "symmetric" (properly: orthogonal) tensor, that is, they are in fact eigendirections. However, principal axes are useful only if the eigendirections are orthogonal. It is necessary to separate the concept of principal axes and that of eigendirections if the latter are non-orthogonal. In such a case principal axes are physically meaningless, but the eigendirections are real and useful. (The significance of eigendirections, and vector space systematics in the modern sense, were fully understood only in 1862, 35 years after Cauchy worked out his stress theory.)

A tensor is a dyadic, a relation between two vectors. In eqn.1 it is the relation between **x** and **b**. In the Euler-Cauchy stress tensor it is the relation between force **f** and the plane orientation vector **n**. Eigendirections are those for which the two vectors involved are parallel. But whereas the eigendirections of eqn.1 may have any angular relation, those of the Euler-Cauchy stress tensor can only be orthogonal – because **n** is by definition orthogonal to the plane. Behind this is Euler's definition of a shear force and normal force being parallel and normal to the plane. This contrasts with Newton's principles where all importance is given to the angular relation between a force **f** and the radius vector **r** of a body. Newton said nothing about the orientation of the surface of the body at the point of action *P* of **f**. **r** is the position vector of *P* relative to the center of mass *Q* of the body, such that *P* and *Q* are different points. In Euler's concept *P* and *Q* are identical. The end point of Euler's **n** is meaningless, it only serves to indicate a direction. Newton's understanding of **r** is fully compatible with eqn.1, Euler's is not; Newton's **r** is a mechanical lever and mathematically identical to **x** in eqn.1, whereas Euler's **n** is not a vector, but a ray (Koenemann 2001T). Cauchy's derivation of the stress tensor must be considered invalid in modern light (Koenemann 2001G, 2008G). This author prefers the term "properties of the loaded state" with the understanding that it is represented by a force vector field.

The quintessential contrast between Eulerian and general field concepts in the sense of eqn.1 may be seen in the perception of a simple shear. Euler's concept led to the concept of a couple, two vectors acting on the same point in a discontinuity, but associated with either side of the discontinuity, and with opposite sense. In geology that would be a fault. But a shear zone has a finite thickness, as in the field given by eqn.4 below; where the shear is a gradient $\partial x_i/\partial x_j$ ($I \neq j$). A couple cannot be described by a field theory because two vectors – let alone with opposite direction – cannot be assigned to the same point. Euler's concept violates the rules for vector spaces (Koenemann 2001T).

The definition of the strain tensor ε is mathematically sound. However, strain is not a thermodynamic state function (Koenemann 2008G). Strain certainly has its use in the geometric analysis of a deformation, but it is not of physical significance. The "instantaneous strain axes" at 45° to the reference frame would be eigendirections; but there is no evidence whatsoever that they exist, they have never been observed; and the fact that the principal axes of finite strain rotate during a simple shear is a clear sign that they are not physically meaningful. Strain is therefore not an adequate conceptual tool to understand the physics of deformation. The natural alternative is the *displacement field*. It is necessary to find the eigendirections of the displacement field which, other than strain, may be orthogonal or monoclinic; they will be delivered by a proper physical approach (Koenemann 2008A).

Earlier concepts of simple shear

The "decomposition" of simple shear

The theorem that governs the understanding of simple shear since the early 19th century is the partition of a general deformation into a translation, a strain and a rigid-body rotation. It is known as the Helmholtz partition or the Cauchy-Stokes decomposition (Truesdell 1954, p.66). The strain is commonly placed at 45° to the reference line X_1 because the asymmetric tensor **D** can be split into a pure shear tensor **S** oriented at 45° to the coordinate axes and a rotation tensor **R**. But this is a misunderstanding.

For ideal simple shear the decomposition **D** = ½ **S** + ½ **R** is

$$a\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \frac{a}{2}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} + \frac{a}{2}\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \qquad \text{(eqn.2)}$$

Geometrically it works – at least it appears to work; but it works only once, not iteratively. The question is both its physical and its geometrical significance. **S** indicates a field with its eigendirections at 45° to the coordinate axes; the contracting eigendirection has a negative slope (from [$-x_1$ x_2] and [x_1 $-x_2$] towards the origin), the extending one has a positive slope (from the origin towards [$-x_1$ $-x_2$] and [x_1 x_2]). **R** is degenerate, and it has imaginary eigendirections. The general rotation matrix is

$$\mathbf{T} = \begin{bmatrix} \cos\gamma & \sin\gamma \\ -\sin\gamma & \cos\gamma \end{bmatrix} \qquad \text{(eqn.3)}$$

Equating **R** = **T** shows that in **R**, γ = 90°. This, and the degenerate and imaginary properties of the characteristic equation of **R** indicate a free, uninhibited spin, but not a rotation by a specific angle.

If a specific strain step *a* is chosen for **S** and the corresponding angle γ is then calculated to find the appropriate **T**, and the operation is repeated in an iterative way, the result will be spurious; the resulting point path is not a simple shear, but it approaches non-orthogonal eigendirections asymptotically. Their orientation plus their mutual angular relations are then a function of the initial chosen strain step *a*. But eigendirections are not for the picking. This, plus the impossibility of a free spin within a bonded continuum, render the partition useless.

The partition would have strong implications on the energetics of deformation because an external rotation does not cost work, hence simple shear and pure shear of the same geometric strain ε should cost the same amount of work. This is not the case (elastic: Treloar 1975, plastic: Franssen & Spiers 1990). Reviewer: download <www.elastic-plastic.de/experimentaldata.pdf>

Any deformation must result in a strain, but nothing is known about the orientation of the finite strain axes relative to a given reference frame without knowledge of the displacement field. But since a pure shear deformation has orthogonal eigendirections, "strain" whose principal axes are orthogonal is often implicitly set equal to a "pure shear strain" which needs then to be rotated externally to give the "shear strain". The result is complete confusion. Pure shear and simple shear are *bulk displacement fields*; strain is a *measure of geometric distortion* only and exclusively, and not a physical term. Strain is not in any way to be mixed up with pure shear. The same restraint should be applied to the concept of shear strain which, like the strain tensor, is a bulk geometric property only and exclusively, without any information on the displacement field or the physical conditions that actually produced it. Strain, displacement and deformation are three terms each with their own meaning; it is wrong to treat them as sub-synonymous.

Just how the partition eqn.2 could assume such importance is curious, because it was never meant to be applicable to solids. The entire early literature is free of any suggestion that there are bonds in continua (Koenemann 2008G). Some authors are perfectly clear in this regard; in particular, Helmholtz (1858) presented the partition as a conceptual tool in a paper on the flow of a friction-free gas – exclusively and explicitly under the rule of the energy conservation law of mechanics E_{kin} + E_{pot} = *const*, by which he clearly stated that he considered flow of a gas as a homonomous flow of discrete particles in freespace, i.e. a conservative flow in which the motion of one particle does not affect the paths of all others; thus interaction between molecules are not electromagnetic, but by collision only. The same understanding is the base of Stokes (1845), and, of necessity, it is implied by all workers prior to Mayer's and Helmholtz' discovery of the First Law of thermodynamics in 1842 and 1847 respectively. However, elastic-reversible deformation of a solid is a change of state in the sense of the

First Law, and viscous flow of fluids or plastic deformation of solids are irreversible, involving the Second Law of thermodynamics. If bonds are not mentioned, they are considered non-existent. The fallacy is not so much in Euler making such an assumption in the 1770s in the attempt to understand the flow of water, rather than carrying it into solids in the late 19th century, and letting it become dogma up to today. The "partition" is incompatible with any conception of both solids and simple shear, and rules out elastic storage of energy, which comes about, after all, through work being done on bonds.

The partition is invalid for other reasons as well. It cannot be given in analytic form, that is, it cannot be expressed as a set of differential equations the integration of which would deliver the desired finite simple shear deformation. Rather than keep guessing along old lines of thought, the right lesson to be drawn is that it is necessary to search for a physical approach that delivers the proper eigendirections as a result. Simple shear and pure shear are not end members of a spectrum, but they are *displacement fields* due to different boundary conditions, requiring different amounts of work.

Ideal simple shear

The transformation

$$\mathbf{Ax} = \mathbf{b} \quad \rightarrow \quad \begin{bmatrix} 0 & a_{12} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ 0 \end{bmatrix} \qquad \text{(eqn.4)}$$

representing an “ideal simple shear” is degenerate; the eigendirections are parallel to x_1 and to one another, but mutually opposite in sense, and det **A** = 0. The field is observed macroscopically in mylonites, or in Couette flow in fluids; but it has never been justified physically. It cannot have physical significance because the condition det **A** = |1| is a powerful conservation law which is clearly not observed. A flow field of this form would imply zero material coherence, which is not even the case in flow of a natural gas under ambient conditions. It does exist phenomenologically – i.e. without any energetic or physical implications – as a bulk flow, but it cannot exist mechanically at the scale at which the deformation mechanism works.

Nature of plastic deformation

Starting from the unloaded standard state, a solid is first loaded elastic-reversibly. An elastic potential builds up, energy is stored in the system. At the yield point the elastic loading stops, and any further work is spent on irreversible processes, such as breaking bonds. The elastic deformation before yield is insignificant in practice, but the elastic state determines the properties of the acting force field, which determines the properties of the displacement field. Upon yield the state of loading remains (to a first approximation) constant. Under the assumption that the configuration of the overall state of loading is undisturbed by microscopic perturbations – e.g. dislocations – the force field which built up during elastic loading, and thus the displacement field, maintain their spatial properties as long as the original external boundary conditions persist. It follows that the eigendirections of the elastically loaded state must be reflected in the fabric of the plastic flow.

Four reasons cause plastic deformation to be heterogeneous by nature.

(a) Elastic work stops at the yield point, any further work goes into plastic-irreversible deformation which is dissipated work. By the law of least work the deformation mechanism which costs less work is favored; hence the volume undergoing continuing plastic deformation will be minimized, and the volumes remaining passively in a constant elastically loaded state will be maximized.

(b) Two types of rotation exist in a bonded continuum undergoing plastic or viscous deformation: shear on a shear zone or a lattice plane, which may be likened to an internal rotation (associated with energy dissipated), and wholesale rotation of a domain relative to the reference frame, which represents an external rotation (associated with energy saved), such that their respective torque cancels (Koenemann 2008A). A plane that glides must also rotate simultaneously, and with opposite sense. Thus flowing continua partition into passively rotating zones and narrow active shear zones such that the torque is balanced.

(c) Bonds cannot be broken by radial compressive loading (by forces colinear to the bond). The only plastic displacement type possible (of a point *P* relative to a reference point *Q*) at the scale at which the deformation mechanism works, is a shear.

(d) Experiments (Franssen & Spiers 1990) and theoretical prediction (Koenemann 2008A) show that plastic simple shear costs ca.30% less work per chosen unit strain than pure shear or axial flattening.

Hence simple shear is expected to be the only deformation type at the scale at which the deformation mechanism works, and any other deformation type is necessarily composite-heterogeneous. The assumption of a perfectly homogeneous plastic or viscous deformation is therefore unrealistic: if bonds are broken, be they permanent as in a solid or transient as in a fluid, the partitioning of a continuum of bonded mass into relatively narrow zones of flow and intermittent zones of relative calm is required by the law of least work. Homogeneous deformation in the plastic field is always a bulk property at a larger scale.

A model for natural plastic simple shear in solids

The following is a synthesis of interpretations based on the recently proposed theory (Koenemann 2008a), and this author's field experience in metamorphic shear zones.

The model

Solids are internally bonded. A solid is in equilibrium with a vacuum, it has a finite volume. It is therefore said to have a nonzero internal pressure; the latter is the pressure that would be observed if a mol of an ideal gas is compressed to the molar volume of the solid; that pressure is internally balanced through the bonds. A change of the volume, or a change of its equilibrium internal structure can only come about through external influences. Bond lengths are changed by work being done by a surrounding upon the system. An elastic deformation is hence a change of state in the sense of the First Law of thermodynamics.

A theory of deformation of solids must therefore start with the First Law and an equation of state; and it must consider the existence of bonds. The latter need to be considered in two different ways: (a) they are the cause of the internal pressure which is a measure of a solid's compressibility; and (b) they have the effect that system and surrounding are solidly bonded to one another. These two points seem to be identical, but systematically they are different since the internal pressure is a natural property of the solid whereas the bonding of system and surrounding is a boundary condition which may or may not hold.

An elastically loaded solid exerts forces at its surrounding, and vice versa. Thus there are two sets of forces: the external forces which are externally controlled by the boundary conditions, and the material forces which are controlled by the material properties. Both can be derived from potentials; the external potential can be postulated, the internal potential is *dU*, or the Helmholtz free energy. The properties of the two force fields are independent of one another; they may indeed be incompatible. Equilibrium is ascertained by the condition that system and surrounding are solidly bonded to one another; the disequilibrium case cannot occur within a solid below the elastic limit. An external acceleration of the system *in toto* is therefore not possible. Thus an external force field of the form of eqn.4 – the external boundary condition – will act upon a system of solid, and interact with the force field exerted by the system (internal boundary condition here: the material is isotropic). The result is a force field with oblique, non-orthogonal eigendirections (Koenemann 2008, fig.11). In the elastic case the force field can be transformed into the displacement field through the work equation; their geometric properties are thus identical. In the plastic case only half the elastic field is commonly realized. This harmonizes with the properties of the energy supply, its boundary conditions (eqn.4; Koenemann 2008A, fig.7), and the nature of plastic flow given above.

The displacement field (fig.1) has a contracting eigendirection *c* inclined against the sense of bulk shear by 111,6°, and an extending eigendirection *e* 10,7° above the bulk shear direction X_1. This indicates that the bulk shear direction X_1 is kinematically nearly dead, it cannot shear. The synthetical maximum shear direction S_{syn} is inclined by 28,8° in the direction of bulk shea r, but it rotates antithetically during flow. The antithetical maximum shear direction is usually not realized, P-plane and synthetical-rotating sector are active around porphyroclasts where the homogeneity of the flow is perturbed, and near the plastic-brittle transition. I correlate S_{syn} with the Riedel shear plane (Riedel 1929) and the C-planes of Berthé et al. (1979); this terminology will be used here, the rationale is given below.

Although the external force field (eqn.4) does not have a component in X_2, both eigendirections of the displacement field are inclined against the reference frame. This appears to be a violation of the boundary conditions, but it is not; the condition is merely that there is no net flow in X_2. The direction *c* forms with X_2 the angle α = 21,63°. Calculated over unit distance in X_2, the vertical inward-flowing component along *c* is proportional to the area ½ tan α = 0,198 = *h*. It is balanced if shortening in X_2 due to flow along *c* and stretch in X_2 due to flow along *e* balance. The X_2-outward component of *e* is tan 10.71°, it is balanced with *h* for a component of *e* in X_1 of 2,097. Hence if the respective flow components were proportional to rock volume, evidence suggesting gentle stretching in X_2 along *e* should be 2,097/0,397 = 5,6 times as common as evidence suggesting strong shortening in X_2 along *c*. The actual volume ratios are different since the work involved differs. The rock volume in the shear zone therefore partitions into subdomains, continuously changing their local flow conditions such that the bulk simple shear field is observed nonetheless, always in such a way that the actively deforming volume is minimized.

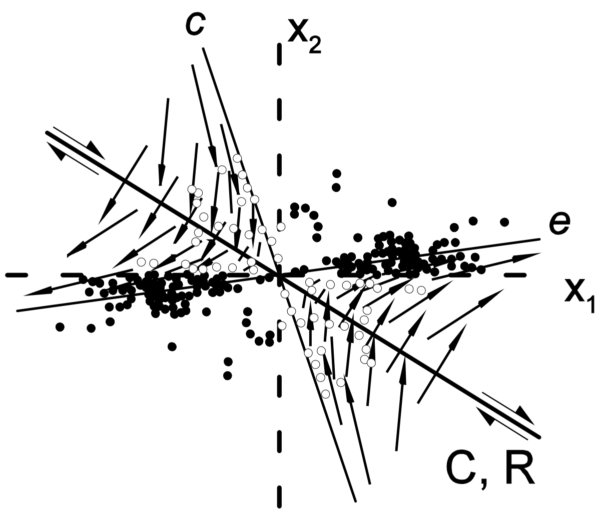

Fig.1 Force field and displacement field for ideal plastic dextral simple shear. *c*, *e*: contracting and extending eigendirections. X_i: external coordinates. C, R: C- or Riedel-plane. Only one shear plane is realized, which is synthetical, and only one set of displacement sectors is realized which causes a volume domainal antithetical rotation, and a stretch. Antithetical volume rotation in X_i and synthetical shear along C balance. The bulk flow plane along X_1 is not an eigendirection of this field. Dots: porphyroclast orientation & aspect ratio data from Law et al. (2004); black dots: forward-rotated σ-clasts, white dots: δ-clasts. Diagnostic clasts shown only.

In fully developed ideal plastic simple shear the shear plane P is commonly suppressed. Synthetical shear takes place on the C-plane (fig.2), thereby defining lozenge-shaped bodies between them which have a shape anisotropy. Initially they behave as passive regions, undergoing antithetical rotation to align their long axis with *e*. Simultaneously they are stretched, and forced to decay by secondary internal C-planes on a smaller scale. The C-planes rotate antithetically out of the maximum shear direction while stretching with the lozenge-shaped bodies between them; they may maintain their identity as zones of concentrated shear deformation until it is energetically easier for a new C-plane to initiate, rather than continue gliding on an old one which has come too close to *e*. As deformation progresses, new generations of C may develop whereas the older ones continuously disappear into the general rock matrix aligning with *e* (cf. White et al. 1980). Deformation starts out heterogeneously and becomes increasingly homogeneous. Two scenarios are then possible: either the rock re-heterogenizes through the development of new C-planes with passive shear bodies in between; or the creation of more pathways changes the material properties which leads to concentration of flow. In either case the process leads to alignment of shape anisotropies of entire shear bodies or the phyllosilicates and fsp-porphyroclasts with *e*.

Concentration of deformation deactivates the shear zone margins; if the displacement rate at large scale is unchanged, more deformation energy is available per unit volume in the regions that continue to deform. The initially coarse fabric of shear bodies switches to smaller scale where the same geometry continues to be active, but the details may be wiped out soon by recovery processes.

Fig.2 Dextral S-C fabric in eclogite, Western Gneiss Region, Norway. Bulk layering inclined ca. 25° to the right, C-planes at 55° offset layering; S-plane s at ca.13° within the shear body mark extending direction *e*. Vertical joints indicate contracting direction *c*. Hammer in black circle for scale.

Nature of compositional banding in mylonites

Thereby the bulk layering, so characteristic of shear zones, develops as a composite fabric at a scale larger than the one on which the deformation mechanism works; it consists at all scales of longer stretches of S-planes and minor parts of C-planes, but it is not indicative of active glide along x_1. The heterogeneous nature of plastic flow never vanishes, it may only be obliterated at small scale by diffusion-controlled processes. It cannot vanish because glide along only one non-rotating plane is physically impossible in a continuum. The bulk fabric in a well-developed mylonite or laminar flow may indeed be understood as an eigendirection – yet not of the physically relevant displacement field (fig.1), but of the resulting bulk flow field (eqn.4) which inherits its properties from the overall boundary conditions. However, it is necessary to recognize them as two different fabric creators. The former is directly controlled by the force/displacement field, it is mechanically relevant and an expression of memory; the latter is mechanically irrelevant and an expression of irreversibility and memory loss.

Domain behavior

One lesson to be drawn from this outline is that there are domains that undergo external rotation instead of internal glide, and commonly this external rotation is antithetical, as in the case of the shear bodies which were created by C-planes cutting through the older layering, or by dissolving a homogeneous (granitic) rock into a particulate tectonite. Thus in the common case that the P-planes are suppressed and C-planes dominate the fabric, the rock volume approaches *e* from below.

The element that is susceptive to reorientation is the anisotropy. It may be the shape anisotropy of non-deforming crystals (tourmaline, fsp, the micas), shear bodies, competent layers that were torn to boudins; it may also be the crystal anisotropy, e.g. for qz in monomineralic layers. For micas the shape anisotropy and the crystal glide plane are identical, which may cause confusion; however, it cannot be the glide plane that causes fabric development since *e* is not a glide plane.

Shear bodies will change shape during progressive deformation due to increasing deformation along their margins. A rule of thumb may be that the more penetrative the deformation is, the softer it is. Shear bodies may therefore change their shape anisotropy during deformation if the ‘erosion’ is not uniform, but affects mostly those parts along the margins which are rotated into active C-planes. Since domain shape anisotropy and crystal anisotropy compete for the position along *e*, it may come to rate-dependent phenomena. Imagine a crystallite deforming plastically by synthetical glide along its lattice planes while simultaneously undergoing external antithetical rotation, but the latter at a faster rate. If the dominating anisotropy is not that of the crystal, but that of the shear body of which it is a part, the crystal glide plane may be rotated beyond *e*. It may continue to glide as long as the external rotation continues, or it may actually reverse the sense of glide. Thus there may be no definite orientation for lattice preferred orientations, only an approximate one depending on local conditions.

C-planes, porphyroclasts, wave crests, and the San Andreas fault

Berthé et al (1979) studied the transformation of a granite which did not offer a natural reference frame. They came to the conclusion that the initial C-panes are parallel to the shear zone boundaries (their fig.3). The same view is taken by Simpson & Schmid (1983), Lister & Snoke (1984) and others. Their interpretation never reflected this author's field experience (some photographic figures in the two last refs appear unduly tilted). Closer to reality are the descriptions by Platt & Vissers (1980) or Dennis & Secor (1987, 1990). More recent studies believe the initial C-plane to be generated at an angle of ca.30° to the shear zone boundaries (Kurz & Northrop 2008). At that angle Riedel (1929) observed shear zones to develop in clay cake experiments.

The obvious inability of S-planes consisting of deactivated layering, or of elongated porphyroclasts (σ-type, Passchier & Simpson 1986) to ever reach the supposed finite strain direction X_1 plus the apparent rotation of C-planes against the bulk sense of shear with progressive deformation were the first incentives for this author to search for a new physical understanding. S-planes are consistently tilted 10° against the bulk sense of shear. They may decay by stretching and be worked into the macroscopic foliation, but this does not remove the 10°-tilt, it only transfers it to a smaller dimension. Coarse σ-type clasts in a fine-grained mylonite are then the remnant of a formerly coarser fabric, still with their long dimension in the 10°-position in which they are apparently stable. The 'stair-step' appearance (Lister & Snoke 1984) is the result of a shear body dissolution having gone to completion until only the clast is left. Their tails merge into the banding along X_1 at the scale of the clast, but this scale is no longer diagnostic for processes in the fine-grained matrix where recrystallization etc. easily erases the structures. The evidence for a stable, non-rotating direction close to 10° above X_1 is therefore strong, is supported by porphyroclast alignment, and compares well with the prediction for *e* (fig.1; Koenemann 2008A, fig.11).

The kinematics of shear zones cannot be anywhere near an "ideal" simple shear. If an apparent homogeneous laminar simple shear flow is observed – in ultramylonites, in fluids – it is due to diffusion-controlled processes which obliterate the traces of the heterogeneous deformation mechanism at a larger scale. δ-porphyroclasts are observed only in ultramylonites (Passchier & Simpson 1986); they are so far the only evidence for whole-body rotation in a shear zone which I can accept. Their generation is not covered by the theory offered here which assumes perfect coherence along grain boundaries. δ-clasts seem to require reduced relative coherence (Fusseis & al. 2009 discovered a considerable porosity in mylonites) and/or strong contrast in material strength between clast and matrix. Reviewer: the porosity in metamorphic mylonites may come as a surprise; check Fusseis et al.

The *c*-direction at 111,6/68,4° is evident in the rock fabric, and ubiquitous in the most various environments. Joints open in this direction either during flow or during uplift (fig.3, 4). The observed maximum compressive stress orientation along the San Andreas fault in California is at 69 ± 14° to the fault (Hickman & Zoback 2004, Boness & Zoback 2006). In porphyroclast-bearing gneisses it is the line that divides σ-clasts with evidence for synthetical rotation from δ-clasts with evidence for antithetical rotation (Simpson & De Paor 1997, fig 9.4: 62°, fig.9.8: 80°; Law et al. 2004, fig.12, and fig.1 this paper: 69°; Kurz & Northrop 2008, fig.4: 72°, 71.5°, 66°, 68°; the original objective of these studies, to determine vorticity numbers [e.g. Passchier 1987], is without physical substance). It is even observed in flowing water (69.4 ± 4.1°, fig.5). Note that the current interpretations vary widely with the scope of the original authors' field of expertise, whereas the listed directions are quite steady and appear to be universal. Reviewer: download <www.elastic-plastic.de/porphyroclasts.pdf>

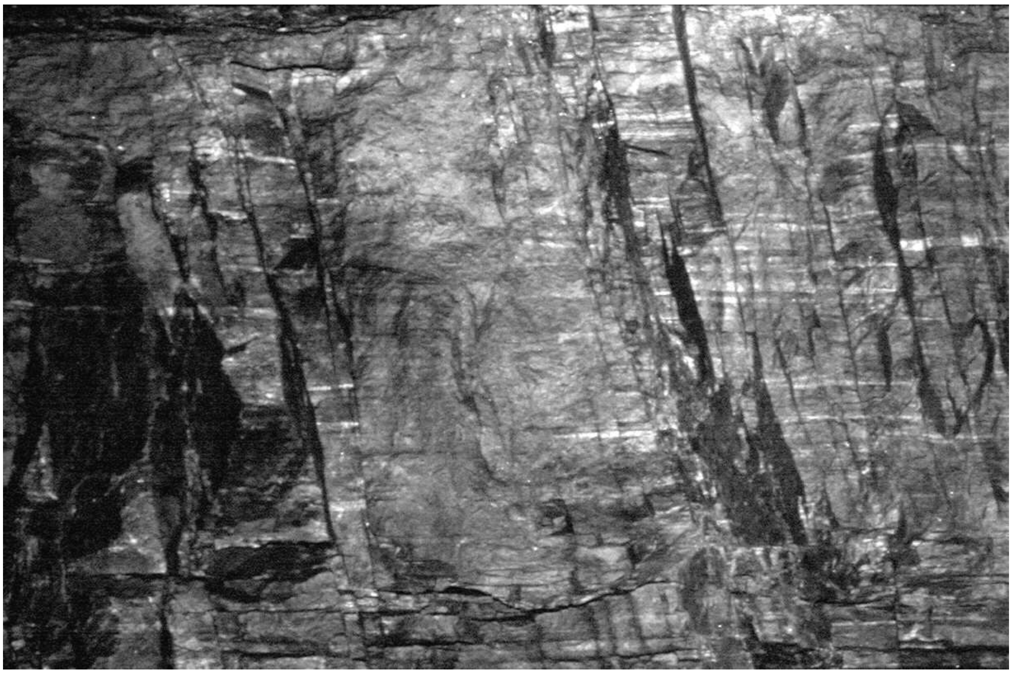

Fig.3 Amphibolite facies mylonite zone, dextral shear, Koralpe, Austria. Joints opened during uplift oriented 105/75 ± 5° to the bulk foliation. View is ca. 1.20 m high.

Fig.4 Viscous dextral simple shear deformation in subrecent obsidian flow, Lipari Island, Italy. Upper layer consists of black glass with abundant vesicles, was softer, and shows drag. Lower layer consists of partly crystallized material, behaved stiffer, and reacted by fracturing.

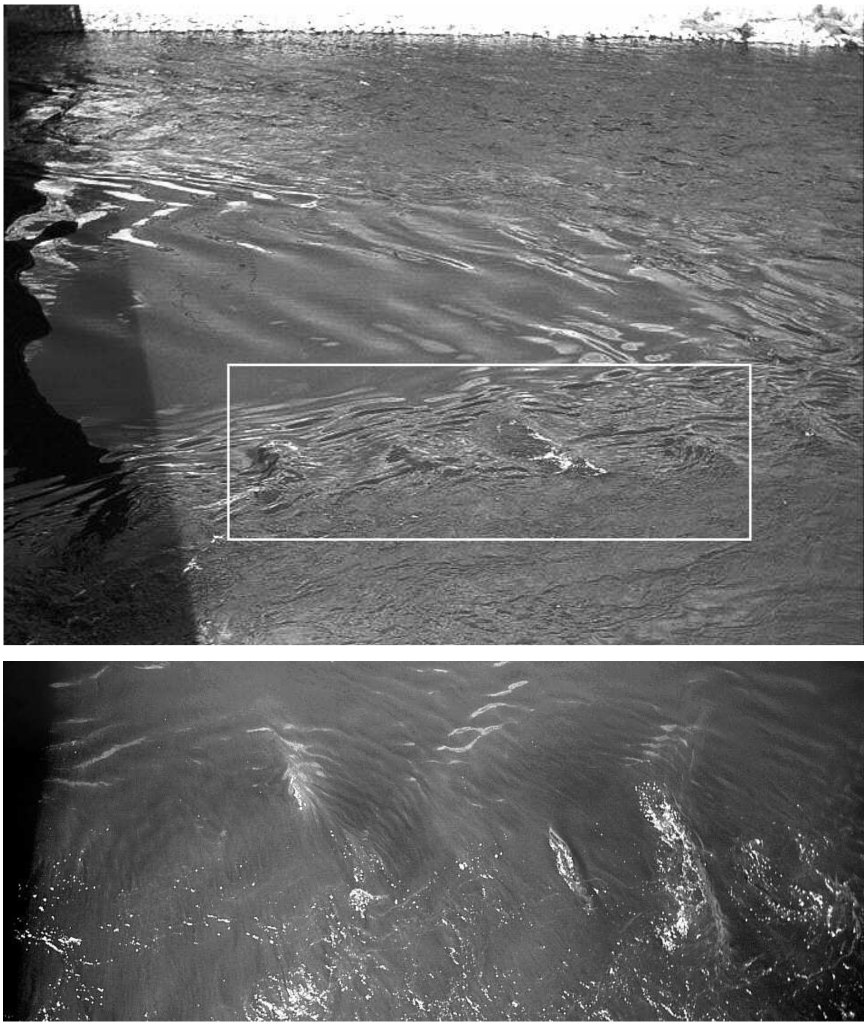

Fig.5 Upper panel: water passing under a bridge flows homogeneously to right at appx. 1 m/sec. Current behind bridge pillar in foreground flows appx. 0.1 m/sec to left (with minor upwelling, mottling the surface), forming a dextral simple shear zone in between. White box, and lower panel: sharply defined standing waves form in the shear zone with average orientation of 69.4 ± 4.1° to SZB (n = 55).

Conclusion

Plastic or viscous flow is irreversible; a flow step can be decomposed into an elastic, reversible, time-independent loading step and a diffusion-controlled, irreversible, time-dependent relaxation step. Homogeneous laminar flow is not possible without relaxation processes, not in rocks and not in fluids. Laminar flow in the form of eqn.4 is a macroscopic phenomenon. At the scale at which the deformation mechanism works, two eigendirections are predicted which can be found in the rock fabric. The flowing rock mass partitions into domains to the effect that the volume undergoing active shear is minimized, and whole-volume rotation is maximized, such that the total deformation work is minimized. Shear and rotation have opposite sign such that the flow is mechanically balanced. At the scale at which the deformation mechanism works, the flow is therefore always heterogeneous. The plastic flow leads to a progressive dissolution of the older rock fabric.

Simple shear flow is stable because it is energetically favored. Its boundary conditions – no net flow perpendicular to the shear zone boundaries – is maintained over the wavelength of an unit distance given by the distance between two C-planes, but local displacements in x_2 do take place. The macroscopic layering parallel to x_1 is the result of the boundary conditions, but it is not a kinematically active eigendirection. It is a composite fabric element consisting of longer parts of S-planes and minor parts of C-planes; these structures are often erased by diffusion-related processes.

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