Prediction of the maximum compression direction along the San Andreas fault and of fabric elements in metamorphic shear zones

Falk H. Koenemann, Im Johannistal 19, 52064 Aachen, Germany; peregrine@t-online.de

Abstract

The maximum compression direction along the San Andreas fault is known to be at 69±14° regionally, and at depth in the SAFOD drill hole, inclined against the sense of shear. A theoretical model predicts a stable direction at 68.4° to the fault. Porphyroclast studies in mylonites revealed a stable direction which divides σ-clasts from δ-clasts. The fabric dividers in recent studies are tightly restricted to angles of 66 to 72°, which is indistinguishable from the predicted 68.4° angle. It is suggested that the two phenomena from the brittle and the plastic field are both expressions of the same cause, the contracting eigendirection of the calculated force/displacement field. Elongated porphyroclasts in mylonites accumulate along a direction ca.10° above the bulk foliation plane, inclined against the sense of bulk shear. The theoretical model predicts a stable direction, the extending eigendirection, at 10.7°. The bisector of the two stable directions is at 28.8° to the bulk foliation and inclined in the sense of bulk shear, it should be a maximum shear direction. This direction is observed in S-C mylonites as the direction of C-plane initiation. It has all the kinematic properties predicted by the model. The theoretical model is therefore fully supported by observations from various fields which have been enigmatic so far.

Introduction

The understanding of stress and deformation is based to this day on a theory that has its roots in the 18th century, and which was worked out by A. Cauchy in 1827-1829. Today it is considered a fairly successful theory; however, this view was not shared unanimously at least in Cauchy's lifetime. *Maxwell* [1850] opened his essay with the statement, "There are few parts of mechanics in which theory has differed more from experiment than in the theory of elasticity"; whereas he mentioned other workers who were known for their experimental experience, he merely referred to 'mathematicians', but would not name Cauchy directly. His statement applies to the handling of the theoretical understanding of elasticity to this day in the sense that the theoretical development is still done mainly by mathematicians, whereas the experimental work is done by material scientists. That is, ultimate authority and experience are not in the same hands. This is certainly not so in thermodynamics.

The theory of elasticity is not in open conflict with experiments for which the boundary conditions have at least orthorhombic symmetry (orthorhombic or axial compression/ stretching). Such experiments were the standard since the formation period of the theory. Simple shear experiments were technically difficult, they are done systematically only since the 1950s when the Cauchy theory had already acquired sacrosanct status. However, with some overview over several fields of research in materials, simple shear deformation seems to be always fraught with unexpected behavior in all four fields of deformation – elastic, viscous, plastic, brittle: solids subjected to plane simple elastic shear expand volumetrically; viscous fluids show laminar flow at slow velocities, but turn to turbulent flow for a reason that is still not understood; plastic simple shear is known to be highly concentrated and appears to be strongly favored, defying the assumption that the stress-strain relation is linear and/or independent of boundary conditions; and the understanding of cracks and joints especially in simple shear leaves much to be desired, starting with the fact that they are usually conjugate, yet the reason is not known. Perhaps Nature thus suggests that the pure or axial deformation state is not diagnostic, whereas the plane simple shear state is critical: it is, after all, the only deformation condition for which the symmetric properties of strain differ from those of displacement; hence simple shear is the pivotal testing ground for deformation theories.

Vector fields were invented by Maxwell in 1861, and the full framework of linear and tensor algebra was worked out by Grassmann in 1878, including the invention of the zero vector and the rules for vector spaces. The modern understanding of classical physics is based on the breakthroughs of the 1850-1880 period, but elasticity is curiously free of them. For example, it is still solidly based on Newton's mechanics. However, elastic deformation is by nature a change of the energetic state in the sense of the First Law of thermodynamics since work is done by a surrounding upon a system, it belongs rightfully into thermodynamics; elastic-reversible work is – for isotropic boundary conditions – PdV-work. There is no hint in the Euler-Cauchy theory that this is so. In none of the many textbooks on the theory of stress and deformation in continuum mechanics which I have seen – such as *Cauchy* [1827a, b], *Love* [1952], Green *and Zerna* [1954], *Truesdell* [1954], *Sneddon* and *Berry* [1958], *Truesdell and Toupin* [1960], *Sokolnikoff* [1964], *Landau and Lifschitz* [1965], *Eringen* [1967], *Malvern* [1968], *Gurtin* [1972, 1981], *Truesdell* [1991], *Holzapfel* [2000] – has there been any mention of the fact that a solid is held together by bonds, at least not in the respective chapters on stress theory, and usually not at all. Bonds do not exist in continuum mechanics. The first author to my knowledge who considered bonds was *Maxwell* [1850]; Cauchy did not yet know about them. Bonds are certainly mentioned in modern engineering textbooks or in fracture mechanics [e.g. *Anderson*, 2005], but the authors are clearly not aware of an incompatibility which nonetheless exists, and which expresses itself in many ways. One cannot ponder a bond-breaking process if bonds are not considered in the theory in the first place. The Euler-Cauchy theory does not even offer a term that can be interpreted to represent bonds, except indirectly through proportionality factors. However, bonds are forces, they need to be considered in the equilibrium equation. – Notably, one single outline to elasticity exists which differs substantially from the Euler-Cauchy theory: the approach by *Helmholtz* [1902; found in July 2010]. It is incomplete due to the author's sudden death in 1894, and subsequently hybridized by his staff who worked his lecture notes into a textbook; apparently they did not recognize the novelty in Helmholtz' thoughts. However, Helmholtz is the only author who derived a vector field from a potential energy term, who made use of a system of unit size, who clearly distinguished system and surrounding as in thermodynamics proper, and who indeed offers a term that can be understood to represent bonds. It is the only genuine precursor to this author's work [*Koenemann*, 2008a].

Natural simple shear

This author's reservations against the Cauchy theory are outlined in *Koenemann* [2001, 2008b]. Instead, thermodynamics in its common form offers a fully satisfying theory of elasticity for a gas subjected to isotropic pressure increase. It also provides the terms that help to understand elastic work as well as a measure of bond strength, the internal pressure $(\partial U/\partial V)_T$. The thermodynamic theory in its common scalar form applies to isotropic conditions, which is sufficient for a gas, but not for a solid. It was thus the intent to transform it into a vector field theory which permits to consider anisotropic boundary conditions as well, and to make it applicable to solids, while leaving the core properties of the scalar theory untouched [*Koenemann*, 2008a]. In this paper I compare several observed aspects of geological simple shear deformation with the predictions derived through the new approach.

The new approach is based on the thermodynamic equilibrium condition $P_{syst} + P_{surr} = 0$. The forces exerted by system and surrounding at one another are derived from two potentials which are differentiated twice with respect to the coordinates to yield two tensors, one for the material properties, one to model the external boundary conditions; the forces $\mathbf{f}_{syst}$ and $\mathbf{f}_{surr}$ are then found through integration over the radius **r** of the thermodynamic system. System and surrounding are thought to be solidly bonded, which has the effect that equilibrium exists by definition as long as no bonds are broken. The result is a force field **f** the properties of which are functions of the material properties and the boundary conditions. The work equation PdV of scalar thermodynamics becomes fdr in vector form; if it is applied to the resulting force vector field **f** the elastic displacement field is derived, such that the resulting force field **f** and the displacement field have identical properties. The strain may then be calculated if desired. The elastic displacements may be minute, but the elastic force field controls the general reaction of the rock to loading below and above the brittle or plastic yield point, and thus the orientation of the developing structures. The force/displacement field for simple shear is shown in Fig.1d. It has a contracting eigendirection (direction of maximum compression) at 68.4/111.6° and an extending eigendirection (direction of tension or minimum compression) at 169.3/10.7° to the fault.

The San Andreas Fault

The stress field along the San Andreas fault in California has been the subject of much research because the maximum compression direction differs significantly from the 45° direction expected so far. Recent results from breakout observations in the SAFOD drill hole have confirmed an orientation of 69±14° at depth in the fault [*Hickman and Zoback*, 2004]; the same direction has been found regionally around the fault [*Zoback et al.*, 1987]. The observed mean direction is indistinguishable from the predicted value (Fig.1a). This author concurs with the common assumption that cracks should open parallel to the maximum loading direction, independent of the theory that is being tested. The prediction is based on the condition that the system is coherent and completely confined, and any free surfaces are infinitely far away. Near an interface to freespace the boundary conditions would change because this would give the material more freedom to relax, which would result in local reorientation of the eigendirections. Measurements at shallow depths are therefore potentially unreliable. The data by *Hickman and Zoback* [2004] are particularly relevant since they were taken at depths where any perturbations in the boundary conditions due to surface proximity or extended cavities along faults in the vicinity can be ruled out.

Fabric properties of mylonites

Passchier and Simpson [1986] found that porphyroclasts in metamorphic shear zones can rotate either way, resulting in structures which they termed σ- and δ-clasts; whereas the σ-clasts indicate a rotation history which is synthetical to the overall sense of shear, the δ-clasts appear to have rotated in an antithetical sense. *Simpson and De Paor* [1997] realized that the σ-δ populations are separated by a line in an orientation diagram which acts as a fabric divider. They also successfully produced two examples in which the divider is oriented at 62/118° and 80/100° to the bulk foliation plane. More recent studies show much less variation; *Law et al.* [2004] found an angle of 69/111°, *Kurz* and *Northrop* [2008] measured angles of 72/108°, 71.5/108.5°, 66/114° and 68/112° (Fig.1b). If taken together, the data concentrate close to 69/111°, with the exception of those of *Simpson and De Paor* [1997]. However, their results are based on relatively few clast observations in comparison to the more recent studies. If their angles are disregarded, the fabric divider is surprisingly stable. The data of *Law et al.* [2004] and *Kurz and Northrop* [2008] form a cluster which is statistically indistinguishable from the 68.4/111.6° orientation for the contracting eigendirection predicted by this author (Fig.1b), with much less spread than the data from the brittle field [*Hickman and Zoback* 2004]. It is therefore suggested that *Simpson and De Paor* [1997] discovered the contracting eigendirection in the mylonitic fabric without using brittle features.

Porphyroclasts with elongated aspect ratios generally show a common orientation in shear zones. By current interpretations they are expected to align with the bulk foliation plane; but this is almost never the case. Instead, *Law et al.* [2004] found that the σ-clast orientations form a cluster ca.10° above the bulk foliation plane, and the larger the aspect ratio, the better the orientations are focused (Fig.1c). The observed direction is again statistically indistinguishable from that of the extending eigendirection at 10.7° predicted by this author. The clast orientations are even more interesting because of their relation to S-C fabrics in mylonites. The bulk foliation in shear zone rocks tends to be composed at the microscopic scale by relatively long parts where the foliation is rotated ca.10° against the sense of shear (the S-plane), and relatively short sections which together form inclined discontinuities on which the shear was synthetical (the C-plane). The latter appear to be generated close to 30° inclined in the sense of shear and are assumed to rotate antithetically during progressive deformation [*Kurz and Northrop*, 2008]. These observations again fully confirm this author's predictions: the bisector of the two eigendirections (see above) at 28,8° below the reference plane (bulk foliation) is expected to be a maximum shear direction; the shear on that plane should be synthetical while the plane simultaneously stretches, and rotates antithetically towards the extending eigendirection (Fig.1d).

Significance for faults and shear zones

The observed data from the brittle and plastic field closely coincide with the eigendirections of the force field, and the elastic displacement field predicted by the new approach [*Koenemann*, 2008a]. As of this point no evidence is known to this author that is not in accord with it; instead, it offers solutions to problems all of which have been without a satisfying explanation for a very long time. There are other aspects of the new approach which are also fully supported by observations, such as the energetics of elastic and plastic deformation in pure and simple shear. The observed 69° direction appears to be a common feature to faults and shear zones in general. It is taken as the maximum compression direction in the brittle field [*Zoback et al*., 1987, *Hickman and Zoback* 2004]. There is no reason to assume a different interpretation in the plastic field. Speculations about vorticity numbers may therefore be misguided. The physics and kinematics of simple shear zones appears to be very different from earlier models. It is possible that a new theoretical understanding of the mechanics of faults is required.

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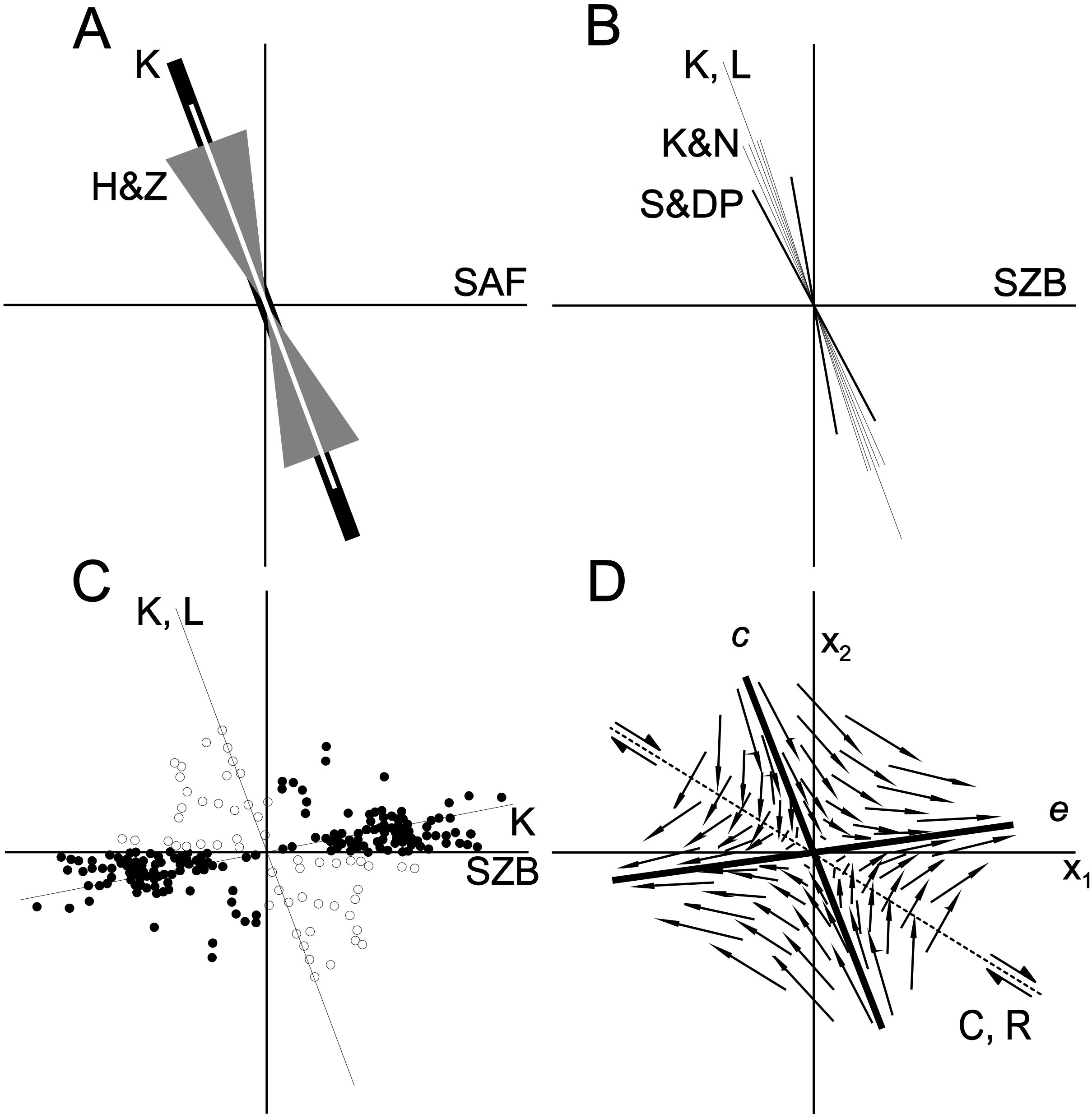

Fig.1: Data and predictions for dextral shear. (A) White, gray [H&Z]: measured maximum compression direction with standard deviation along the San Andreas fault (SAF) at 69/111±14° [*Hickman and Zoback*, 2004]. Black [K]: predicted direction at 68.4/111.6° [*Koenemann*, 2008a]. (B) Fabric divider lines from clast orientation studies in mylonites. Short lines [S&DP]: *Simpson and De Paor* [1997]; medium lines [K&N]: *Kurtz and Northrop* [2008]; long line [K, L]: Law et al. [2004], which is indistinguishable from prediction by *Koenemann* [2008a]. SZB: shear zone boundary or bulk foliation plane. (C) Porphyroclast orientation and aspect ratio data [*Law et al.*, 2004]; black dots: forward-rotated σ-clasts, open dots: δ-clasts. Diagnostic clasts shown only. Fabric divider line [L] at 69/111° as in (B); long axes of σ-clasts accumulate along a direction ca.10° above the SZB. Predicted directions [K] as in (D). (D) Predicted force/displacement field for homogeneous elastic-reversible simple shear [*Koenemann*, 2008a]. *c*: contracting eigendirection at 68.4/111.6°; *e*: extending eigendirection at 169.3/10.7°; C, R: C-plane in SC-fabric, and Riedel plane in plastic deformation.